TRINITY COLLEGE

Mathematics Specialist Units 3 & 4
Test 1   2016

Section 1   Calculator Free

Complex Numbers

STUDENT'S NAME: ____

DATE: Thursday 5th November   **TIME:** 25 minutes   **MARKS**: 30

INSTRUCTIONS:

Standard Items: Pens, pencils, pencil sharper, eraser, correction fluid/tape, ruler, highlighters, Formula Sheet.

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (6 marks)

Convert $\frac{-1+i\sqrt{3}}{2}$ to polar form and hence evaluate $\left(\frac{-1+i\sqrt{3}}{2}\right)^8$, giving your result in Cartesian form $a+bi$.

2. (8 marks)

Solve the following equations:

(a) $3z^2 + 3z + 1 = 0$ [3]

(b) $5z^3 - 12z^2 + 5z - 2 = 0$ [5]

3. (8 marks)

Solve the following equations, stating the roots in polar form and showing them on an Argand diagram:

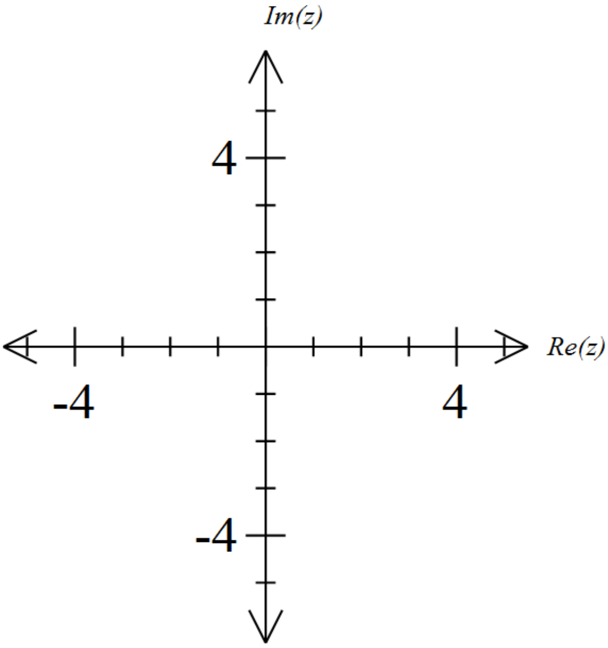
(a) $z^6 = 1$ [4]

(b) $z^3 - 64i = 0$ [4]

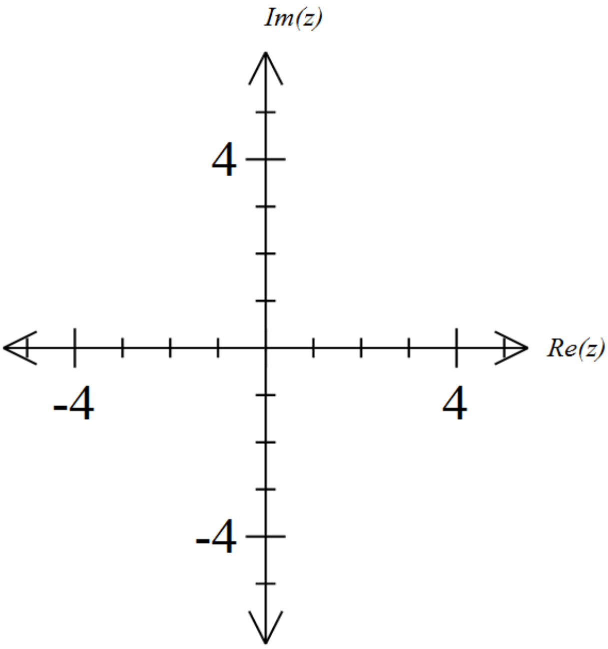
4. (8 marks)

Given that $a = 3 + 2i$ and $b = -1 + 2i$. Clearly label the set of points on each Argand diagram defined by:

(a) $|z - b| < 2$ [4]



(b) $|z + a| \leq |z + b|$ [4]



End of Questions

TRINITY COLLEGE

Mathematics Specialist Units 3 & 4
Test 1   2016

Section 2   Calculator Assumed

Complex Numbers

STUDENT'S NAME: ____

DATE: Thursday 5th November   **TIME:** 25 minutes   **MARKS**: 30

INSTRUCTIONS:

Standard Items: Pens, pencils, pencil sharper, eraser, correction fluid/tape, ruler, highlighters, Formula Sheet retained from Section 1.

Special Items: Drawing instruments, templates, three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment).

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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5. (5 marks)

Use your calculator to:

(a) Convert $\dfrac{-1+i\sqrt{3}}{2}$ to polar form. [1]

(b) Evaluate $\left(\dfrac{-1+i\sqrt{3}}{2}\right)^8$, giving your result in Cartesian form $a+bi$. [2]

(c) Solve $5z^3-12z^2+5z-2=0$ [2]

6. (10 marks)

(a) Are the following statements True or False? [4]

(i) $\text{cis}(\pi) = -1$

(ii) $\arg(z^{-1}) = -\arg(z)$

(iii) $|z^n| = |z|^n \quad \forall n \in \mathbb{Z}$

(iv) $(\text{cis}\,\theta)^n = \text{cis}(n\theta) \quad \forall n \in \mathbb{Z}$

(b) State the conditions under which the following statements are true: [3]

(i) If $z_1 = x_1 + y_1 i$ and $z_2 = x_2 + y_2 i$ then $x_1 = x_2$ and $y_1 = y_2$

(ii) If $z_1 = r_1 \text{cis}\,\theta_1$ and $z_2 = r_2 \text{cis}\,\theta_2$ then $r_1 = r_2$ and $\theta_1 = \theta_2$

(iii) $z^{-1} = \bar{z}$

(c) Given $z = |z|\,\text{cis}\,\theta$, prove $z\bar{z}$ is always purely real. [3]

7. (8 marks)

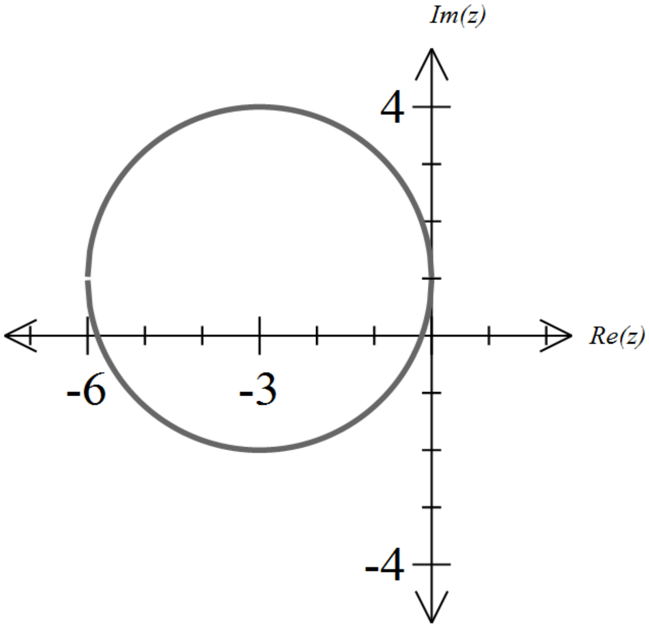
(a) Prove for $z \in \mathbb{C}$, $z^{-1} = \dfrac{\bar{z}}{|z|^2}$ [4]

(b) If $z = \text{cis}\,\theta$, simplify $z - \dfrac{1}{z}$. [4]

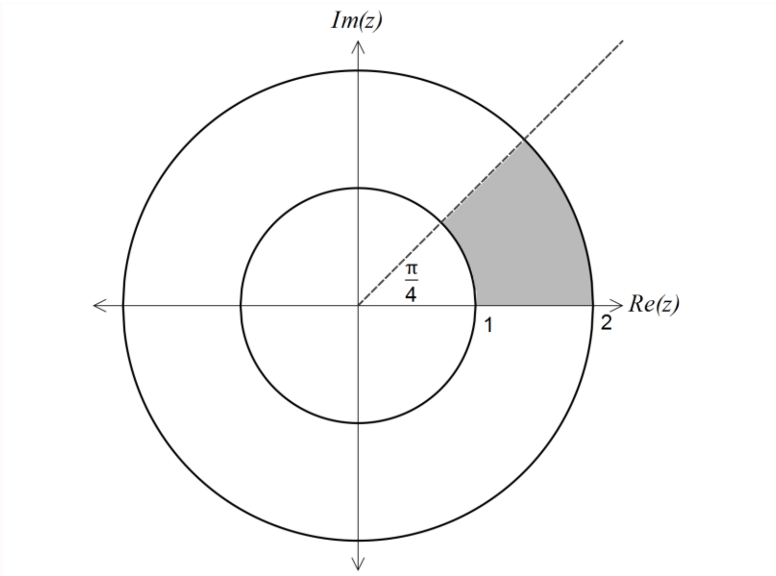
8. (7 marks)

Describe, using appropriate notation, the following sets of points:

(a) [3]



(b) [4]



End of Questions